UNIT # 03 (PART – I)
CENTRE OF MASS

EXERCISE –I

1. $\bar{x} = \dfrac{\int x\,dm}{\int dm} = \dfrac{\int_0^L x \dfrac{kx^2}{L} dx}{\int \dfrac{kx^2}{L} dx} = \dfrac{3L}{4}$

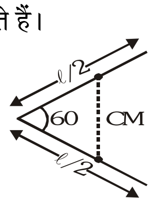
2. Equation of line joining the CM of two rods
दोनों छड़ों के द्रव्यमान केन्द्र को मिलाने वाली रेखा का समीकरण

$$\frac{x}{L/2} + \frac{y}{L/2} = 1$$

coordinate $\left(\dfrac{L}{3}, \dfrac{L}{6}\right)$ satisfies this equation.

निर्देशांक $\left(\dfrac{L}{3}, \dfrac{L}{6}\right)$ समीकरण को संतुष्ट करते हैं।

3. $\bar{x} = \dfrac{\ell}{4}\cos 30° = \dfrac{\sqrt{3}\ell}{8}$



4. Let x_p=x shift of plank to the right
माना x_p=x तख्ते का दांयी ओर विस्थापन

$$\Delta\bar{x} = \frac{m_A \Delta x_A + m_B \Delta x_B + m_c \Delta x_c + m_p \Delta x_p}{m_A + m_B + m_C + m_P}$$

$$0 = \frac{40(x+4) + 50x + 60(x-4) + 90x}{40+50+60+90} \Rightarrow x = \frac{1}{3}\text{ m}$$

5. $\Delta\bar{x} = \dfrac{m_1.\Delta x_1 + m_2.\Delta x_2}{m_1 + m_2}$

$$\Rightarrow 0 = \frac{m_1 a + m_2 \Delta x_2}{m_1 + m_2} \quad \Rightarrow \Delta x_2 = -\frac{m_1 a}{m_2}$$

6. $\Delta\bar{y} = \dfrac{m_1 \Delta y_1 + m_2 \Delta y_2}{m_1 + m_2}$

$$\Rightarrow 0 = \frac{m}{4}(15) + \frac{3m}{4}(y_2) \Rightarrow y_2 = -5 \text{ cm}$$

7. CM remains at rest if initially it is at rest.
द्रव्यमान केन्द्र स्थिर बना रहता है यदि प्रारम्भ में यह विरामावस्था में है।

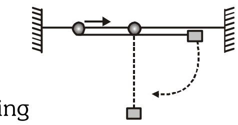
8. $\vec{v}_{cm} = \dfrac{m_1\vec{v}_1 + m_2\vec{v}_2}{m_1 + m_2} = \dfrac{1 \times 2\hat{i} + 2 \times (2\cos 30\hat{i} - 2\sin 30\hat{j})}{3}$

$$= \left(\frac{2 + 2\sqrt{3}}{3}\right)\hat{i} - \frac{2}{3}\hat{j}$$

9. $v_{CM} = \dfrac{(1)(5) + (1)(-3)}{1+1} = 1 \text{ m/s}$
Position of centre of mass at t=1s
t = 1s पर द्रव्यमान केन्द्र की स्थिति

$$X_{CM} = \frac{(1)(2) + (1)(8)}{1+1} + (1)(1) = 5 + 1 = 6\text{m}$$

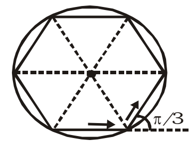
10. $\Delta\bar{x} = \dfrac{m_1.\Delta x_1 + m_2.\Delta x_2}{m_1 + m_2}$



Let x = distance moved by ring
माना x = वलय द्वारा चली गई दूरी

$$0 = \frac{mx + 2m(1.2 - x)}{m + 2m} \Rightarrow x = 0.8 \text{ m}$$

11. Impulse (आवेग)
$= p_f - p_i = 1 \quad 10 - 1 \quad (-25) = 35\text{kg m/s} (\uparrow)$

12. 

$\Delta p = 2p\cos\dfrac{\pi}{3} = 2mv_0 \sin\left(\dfrac{\pi}{6}\right)$

13. For the I[st] ball (प्रथम गेंद के लिए): $\dfrac{h}{4} = e_1^2 h$

For the II[nd] ball (द्वितीय गेंद के लिए): $\dfrac{h}{16} = e_2^2 h$

Impulse on first ball (प्रथम गेंद पर आवेग)

$$= I_1 = mv_0(1+e_1) = \frac{3}{2}mv_0$$

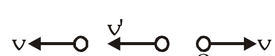
Impulse on second ball (द्वितीय गेंद पर आवेग)

$$= I_2 = mv_0(1+e_2) = \frac{5}{4}mv_0$$

$$\Rightarrow \frac{I_1}{I_2} = \frac{3/2\ mv_0}{5/4\ mv_0} = \frac{6}{5} \Rightarrow 5I_1 = 6I_2$$

14. $\Delta KE = \dfrac{1}{2}mv_2^2 - \dfrac{1}{2}mv_1^2$

$$= \frac{1}{2}m(\vec{v}_2 - \vec{v}_1).(\vec{v}_2 + \vec{v}_1) = \frac{1}{2}\vec{I}.(\vec{v}_1 + \vec{v}_2)$$

15. 

0= 2mv − mv + mv' ⇒ v' = −v
Total mechanical energy released
(मुक्त हुई कुल यांत्रिक ऊर्जा)

$$= \frac{1}{2}(m+m+2m)v^2 = 2mv^2$$

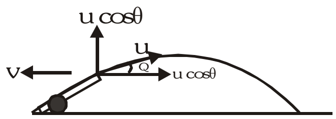
16. COLM : $3 \quad 2 = (3+2)v \Rightarrow v = \frac{6}{5}$ m/s

COME : $\frac{1}{2}\times 3\times 2^2 = \frac{1}{2}\times 5\times\left(\frac{6}{5}\right)^2 + \frac{1}{2}\times 480\times x^2$

$\Rightarrow x = \frac{1}{10}$m

17. COLM :

Along horizontal (क्षैतिज के अनुदिश)



$0 = m(u\cos\theta - v) - 4mv \Rightarrow v = \frac{u\cos\theta}{5}$

$\therefore$ velocity of shell along horizontal w.r.t ground (धरातल के सापेक्ष क्षैतिज के अनुदिश गोले का वेग)

$= u\cos\theta - \frac{u\cos\theta}{5} = \frac{4}{5}(u\cos\theta)$

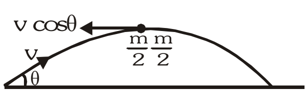
Time of flight (उड्डयनकाल) $T = \frac{2u\sin\theta}{g}$

$\therefore$ x = horizontal displacement (क्षैतिज विस्थापन)

$= \left(\frac{4}{5}u\cos\theta\right)\left(\frac{2u\sin\theta}{g}\right) = \frac{4u^2\sin 2\theta}{5g}$

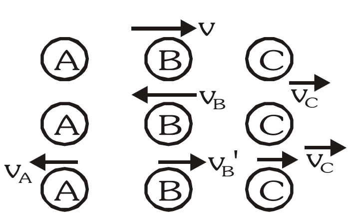
18. COME : $mv\cos\theta = \frac{m}{2}(-v\cos\theta) + \frac{m}{2}v'$

$\Rightarrow \frac{3mv\cos\theta}{2} = \frac{m}{2}v' \Rightarrow v' = 3v\cos\theta$



19. The ball & the earth froms a system and no external force acts on it. Hence total momentum remains constant. (निकाय से पृथ्वी तथा गेंद पर कोई बाह्य बल नहीं लगता है अत: कुल संवेग नियत बना रहता है)

20.



For collision between B and C :
(B तथा C के मध्य टक्कर के लिए)

$v_B = \left(\frac{m_1 - em_2}{m_1+m_2}\right)u_1 + \frac{m_2(1+e)}{(m_1+m_2)}u_2$

$= \left(\frac{m-4m}{5m}\right)v + 0 \quad = -3/5\ v$

$v_C = \frac{m_1(1+e)}{(m_1+m_2)}u_1 + \left(\frac{m_2 - em_1}{m_1+m_2}\right)v_2 = \frac{m(1+1)}{4m}v + 0 = \frac{v}{2}$

For collision between A and B :
(A तथा B के मध्य टक्कर के लिए)

$v_A = 0 + \frac{m(1+1)}{5m}\times\left(\frac{3v}{5}\right) = \frac{6}{25}v$

$v_B' = 0 + \left(\frac{m-4m}{5m}\right)\left(-\frac{3v}{5}\right) = -\frac{9v}{25}$

$\because v_B' < v_C$

$\therefore$ B will not collide with C.
(अत: गेंद B, गेंद C से नहीं टकरायेगी)

Therefore there will be only two collisions.
(इसलिए दोनों के बीच केवल दो टक्कर होगी)

21. COME : $m_1u_1 + m_2u_2 = m_1v_1 + m_2v_2$

$1 \quad u + 0 = 1 \quad \frac{u}{4} + mv_2 \Rightarrow \frac{3}{4}u = mv_2$ ....(i)

$e = -\left(\frac{v_2 - v_1}{u_2 - u_1}\right) = -\left(\frac{v_2 - u/4}{0-u}\right) = \frac{v_2 - u/4}{u}$

$v_2 = \frac{u}{4} + u = \frac{5u}{4}$ ....(ii)

(i) & (ii) $\frac{3}{4}u = m\left(\frac{5}{4}u\right) \Rightarrow m = \frac{3}{5} = 0.6$kg

22. At the lowest position (निम्नतम स्थिति पर)

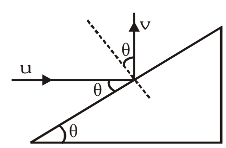
COME : $M\sqrt{2gL} = (M+m)v$ ....(i)

COME : $\frac{1}{2}(M+m)v^2 = (M+m)gh$ ....(ii)

$\Rightarrow v = \sqrt{2gh} = \frac{M\sqrt{2gL}}{(M+m)} \Rightarrow h = \left(\frac{M}{m+M}\right)^2 L$

23. Along tangent (स्पर्श रेखा के अनुदिश)

$u\cos\theta = v\sin\theta$...(i)

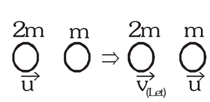


Along normal (अभिलम्ब के अनुदिश)

$e = \frac{v\cos\theta}{u\sin\theta} = \cot^2\theta = \cot^2 60 = \frac{1}{3}$

24. COLM $\Rightarrow$ 2mu + 0= 2mv + mu $\Rightarrow v = \frac{u}{2}$

$e = -\left(\frac{v_2 - v_1}{u_2 - u_1}\right) = -\left(\frac{u - u/2}{0-u}\right) = \frac{1}{2}$



25. After 1s , $v_A = 20–10 \quad 1 = 10$ m/s

and $v_B = 0 + 10 \quad 1 = 10$ m/s

At the time of collision (टक्कर के समय)

, $V_A = V_B = 5$m/s

after collision, velocity gets enterchanged.

(टक्कर के पश्चात्, वेग आपस में बदल जाते हैं)

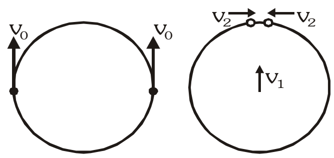
26. $e = -\left(\dfrac{v_2 - v_1}{u_2 - u_1}\right) \Rightarrow 1 = -\dfrac{5 - v_1}{5-(-10)} = \dfrac{v_1 - 5}{15}$

$\Rightarrow v_1 = 20$ m/s

$\therefore$ Impulse on ball= $m(\vec{v} - \vec{u})$ =1 [20–(10)]=30N-s

(गेंद पर आवेग)

27.



COLM $\Rightarrow$ $2mv_0 = 3mv_1 \Rightarrow v_1 = \dfrac{2}{3}v_0$

COME $\Rightarrow$ $2 \times \dfrac{1}{2}mv_0^2 = 2 \times \dfrac{1}{2}m(v_1^2 + v_2^2) + \dfrac{1}{2}mv_1^2$

$\Rightarrow 2mv_0^{\,2} = 2m\left(\dfrac{2}{3}v_0\right)^2 + 2mv_2^2 + m\left(\dfrac{2}{3}v_0\right)^2$

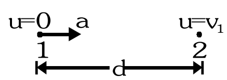
$\Rightarrow 2v_2^2 = 2v_0^2 - 3\left(\dfrac{2}{3}v_0\right)^2 \Rightarrow v_2 = \dfrac{v_0}{\sqrt{3}}$

Velocity of particle = $\sqrt{v_1^2 + v_2^2} = \sqrt{\dfrac{4v_0^2}{9} + \dfrac{v_0^2}{3}} = \dfrac{\sqrt{7}}{3}v_0$

(कण का वेग)

28. For second object (दूसरी वस्तु के लिए)

$2v_1 = 0 + at;\ t = \dfrac{2v_1}{a}$ ...(i)



$s_1 = s_2 + d;\ \dfrac{1}{2}at^2 = ut + d$

$\Rightarrow \dfrac{1}{2}a \times \left(\dfrac{2v_1}{a}\right)^2 = v_1\left(\dfrac{2v_1}{a}\right) + d \Rightarrow d = \dfrac{2v_1^2}{a} - \dfrac{2v_1^2}{a} = 0$

29. Average power (औसत शक्ति)

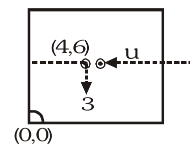
$\dfrac{\Delta W}{\Delta t} = \left(\dfrac{\Delta K + \Delta U}{\Delta t}\right) = \dfrac{1}{2}\dfrac{(\lambda \Delta x)v^2}{\Delta t} + \dfrac{\lambda \Delta x g(\Delta x/2)}{\Delta t}$

$\langle P \rangle = \dfrac{1}{2}\lambda v^3 + \dfrac{\lambda \ell}{2}vg$

EXERCISE –II

1. In the absence of external forces, the linear momentum of the system remains constant.
(बाह्य बलों की अनुपस्थिति में निकाय का रेखीय संवेग नियत बना रहता है)

2. Momentum of the coin perpendicular to the common normal remains constant. (सिक्के का संवेग उभयनिष्ठ अभिलम्ब के लम्बवत् नियत होता है)



$\therefore v_y = -3$ (constant)

$v_y t = -6 \Rightarrow t = 2$ sec & $v_x t = -4$

$v_X = -2$ m/s

Which is given by stricker.

(जो कि स्ट्राईकर द्वारा दिया जाता है)

So initial velocity of stricker = $2ms^{-1}$

(अतः स्ट्राईकर का प्रारम्भिक वेग)

Final velocity of the striker = 0.

(स्ट्राईकर का अन्तिम वेग)

3. $F_{ext} = m\dfrac{dv}{dt} + v_{rel}\dfrac{dm}{dt} \Rightarrow 0 = m\dfrac{dv}{dt} + 2\dfrac{dm}{dt}$

$\Rightarrow -\displaystyle\int_{+m}^{m/2}\dfrac{2dm}{m} = \int_0^v dv \Rightarrow v = 2\ell n2$

4. In the ground frame (जमीनी तंत्र में)

: $m_A\Delta x_A + m_B\Delta x_B + m_p\Delta x_p = 0$

$\Rightarrow 40 \quad 60 + 0 + 40 \quad \Delta x_p = 0$

$\Rightarrow \Delta x_p = -60$ (to the left)

Hence A & B meet at the right end.

(अतः A व B दांये सिरे पर मिलते हैं)

5. Force exerted by one leg on the ground

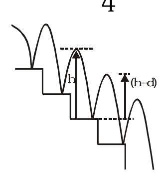
(मेज की एक टांग द्वारा जमीन पर लगाये जाने वाला बल)

N $= \dfrac{1}{4}$ [Total force] कुल बल

$= \dfrac{1}{4}$ [wt + rate of change of momentum]
[wt + संवेग में परिवर्तन]

$= \dfrac{1}{4}$[Mg + n (mv cos 60) 2]= 1N

6.

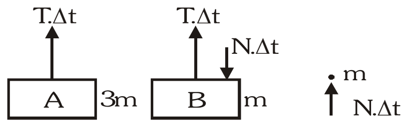


$\sqrt{2g(h-d)} = e\sqrt{2gh} \Rightarrow \dfrac{h}{d} = \dfrac{1}{1-e^2}$

7. COLM : $m_R(0.8) + m_S(0) = m_R(0.2) + m_S(1.0)$

$\Rightarrow 0.6\ m_R = m_S \Rightarrow m_R > m_S$

8.



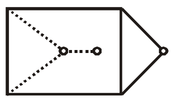
$-N\Delta t = m(v-u)$; $N\Delta t - T\Delta t = m(v-0)$

$T\Delta t = 3m(v-0) \Rightarrow v = u/5$

$\therefore$ Impulsive tension (आवेगीय तनाव)$T\Delta t = \dfrac{3mu}{5}$

9. $\bar{x} = \dfrac{M_1x_1 + M_2x_2}{M_1+M_2} = \dfrac{\frac{M}{2}\left(-\frac{a}{2}\right)+\frac{M}{2}\left(\frac{a}{3}\right)}{M} = -\dfrac{a}{12}$

10.

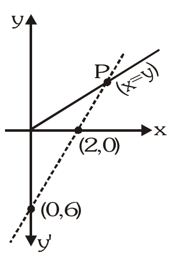


$$\bar{x} = \frac{(M\times 0)+\left(-\frac{M}{4}\times\frac{-L}{3}\right)+\left(\frac{M}{4}\times\frac{4L}{6}\right)}{M} = \frac{L}{4}$$

11. $\Delta\bar{x} = \dfrac{3M.x + M(x+2)}{4M} = 0 \Rightarrow x = -\dfrac{1}{2}$

12. $Nmv = (M + Nm)v_f \Rightarrow v_f = \dfrac{mvN}{M+Nm}$

13.



$\dfrac{x}{2} - \dfrac{y}{6} = 1$

Co-ordinate of P (P के निर्देशांक)= (3,3)

$\therefore$ Speed of 3rd particle (3rd कण की चाल)= $3\sqrt{2}$ m/s

14. Let x = displacement of ring to the left.
(माना x=बांये ओर वलय का विस्थापन)

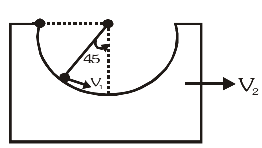
$\Rightarrow \Delta x_{cm} = \dfrac{2mx + m(x + L - L\cos\theta)}{3m} = 0$

$\Rightarrow x = -\dfrac{L}{3}(1-\cos\theta)$

15. $Mv = 0 + \dfrac{M}{10}v_2 \Rightarrow v_2 = 10v$

16. COLM $\Rightarrow m(v_1 \cos 45 + v_2) + mv_2 = 0$

$\Rightarrow v_1 = -2\sqrt{2}.v_2$



COME $\Rightarrow K_i + U_i = K_f + U_f$

$\Rightarrow 0 + \dfrac{mgR}{\sqrt{2}} = \dfrac{1}{2}mv_2^2$

$+\dfrac{1}{2}m\left[\dfrac{v_1^2}{2}+\left(\dfrac{v_1}{\sqrt{2}}-\dfrac{v_1}{2\sqrt{2}}\right)^2\right] \Rightarrow v_2 = \sqrt{\dfrac{gR}{3\sqrt{2}}}$

17. Initially when the shell is empty the C.M. lies at its geometric centre. Also when the shell is filled with sand CM lies at its geometric centre.
(प्रारम्भ में जब गोलीय कोश रिक्त है तो द्रव्यमान केन्द्र इसकी ज्यामितीय केन्द्र पर स्थित होगा तथा कोश को रेत से पूरा भर दिया जाये तो द्रव्यमान केन्द्र इसके ज्यामितीय केन्द्र पर स्थित होगा)

18. $a_{cm} = \dfrac{F_{net}}{\text{Total mass}} = \dfrac{(0.2)(3)(10)}{1+2} = 2\,ms^{-2}$

OR

Acceleration of 1kg w.r.t. ground
(जमीन के सापेक्ष 1kg का त्वरण)

$=(0.1)(10)=1ms^{-2}$

Accceleration of 2 kg w.r.t. ground
(जमीन के सापेक्ष 2kg का त्वरण)

$= \dfrac{(0.2)(3)(10)-(0.1)(10)}{2} = \dfrac{5}{2}ms^{-2}$

$a_{cm} = \dfrac{m_1a_1+m_2a_2}{m_1+m_2} = \dfrac{(1)(1)+(2)(5/2)}{1+2} = 2ms^{-2}$

19. Δp = change in momentum (संवेग में परिवर्तन)= 2mv

Δt = time between two collision $= \dfrac{2(L-d)}{v}$

(दो टक्करों के मध्य समय)

$\therefore$ Force exerted on wall (दीवार पर आरोपित बल)

$= \dfrac{\Delta p}{\Delta t} = \dfrac{mv^2}{(L-d)}$

20. $t_1 = \dfrac{L}{v}$ (time for Ist collision) पहली टक्कर के लिए समय

$t_2 = \dfrac{2L}{v}$ (time for IInd collision)
द्वितीय टक्कर के लिए समय

$t_3 = \dfrac{3L}{v}$ (time for 3rd collision)
तीसरी टक्कर के लिए समय

$t_{(n-1)} = \dfrac{L}{v}(n-1)$ (time for (n^{th}) collision)
nवीं टक्कर के लिए समय

$\sum_{i-1}^{h} t_i = \dfrac{n(n-1)}{2}\dfrac{L}{v}$

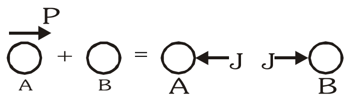
21. In elastic head on collision if the masses of the colliding bodies are equal, the velocities after collision are interchanged.(सम्मुख प्रत्यास्थ टक्कर में यदि टक्कर कर रही वस्तुओं के द्रव्यमान समान हैं तो टक्कर के बाद उनके वेग आपस में बदल जाते हैं)

For Ist bead(मोती), $Fd = \dfrac{1}{2}mu^2 \Rightarrow u = \sqrt{\dfrac{2Fd}{m}}$

22. $\Delta\bar{x} = \dfrac{m_1\Delta x_1 + m_2\Delta x_2 + m_3\Delta x_3}{m_1 + m_2 + m_3}$

$\Delta\bar{x} = \dfrac{(80\times 2)-(50\times 2)+(70\times 0)}{80+50+70}$

= 30cm towards right

23. 

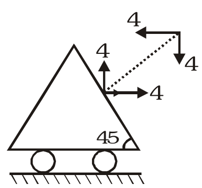
For A : $P - J = mv_1$ ....(i)

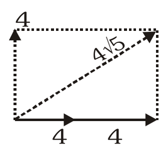
for B : $J = mv_2$ ....(ii)

$\therefore e = -\left(\dfrac{v_2 - v_1}{u_2 - u_1}\right) = -\left[\dfrac{\dfrac{J}{m} - \left(\dfrac{P-J}{m}\right)}{0 - \dfrac{P}{m}}\right] = \dfrac{2J}{p} - 1$

24. $mu - I_1 = -mu \;\therefore\; I_1 = 2mu$ &

$mu - I_2 = 0 \Rightarrow I_2 = mu$ (for II[nd] ball) $\therefore I_2 = I_1/2$

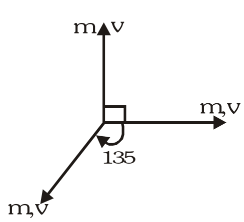
25. Cart frame (गाड़ी तंत्र): 

Ground frame (जमीनी तंत्र): 

The velocity of rebound (टकराने का वेग) = $4\sqrt{5}$ m/s

26. COLM $\Rightarrow$ $mv\hat{i} + mv\hat{j} + m\left(\dfrac{-v}{\sqrt{2}}\hat{i} - \dfrac{v}{\sqrt{2}}\hat{j}\right) + m\vec{v}_4 = 0$

$\vec{v}_4 = -v\left(1 - \dfrac{1}{\sqrt{2}}\right)\hat{i} - v\left(1-\dfrac{1}{\sqrt{2}}\right)\hat{j}$



Total energy released (मुक्त हुई कुल ऊर्जा)

$= \dfrac{1}{2}mv^2 + \dfrac{1}{2}mv^2 + \dfrac{1}{2}mv^2 + \dfrac{1}{2}m\left[v^2\left(1-\dfrac{1}{\sqrt{2}}\right)^2 \times 2\right]$

= $mv^2(3-\sqrt{2}\,)$

27. $F_{ext} = \Delta mv$ (for first body)$\Rightarrow F_{ext} = \dfrac{10\times(15-0)}{3} = 50N$

COME : $m_1u_1 + m_2u_2 = (m_1+m_2)v$

$\Rightarrow 10 \quad 15 + 25 \quad u_2 = (10 + 25)5 \Rightarrow u_2 = 1$ m/s

28. COME : $-MV + m(v\cos 60 - V) = 0 \Rightarrow v = 10$ m/s

29. $\Delta\bar{x} = \dfrac{m\Delta x_1 + M\Delta x_2}{m+M} = 0$

$\Rightarrow \dfrac{1(\ell\sin 30° + \ell\sin 30° - x) + 4x}{1+4} = 0$

$\Rightarrow$ Displacement of bar (छड़ का विस्थापन)= x = 0.2

30. $x_{cm} = \dfrac{m\times 0 + m\times R}{m+m} = \dfrac{R}{2}$

$x_{CG} = \dfrac{W_1(0) + W_2(R)}{W_1 + W_2} = \dfrac{mgR}{mg} = R$

$\therefore x_{CG} - x_{CM} = R - \dfrac{R}{2} = \dfrac{R}{2}$

31. Velocity before strike (टकराने के पहले वेग)

$u = \sqrt{2gh}$

Impulse (आवेग) = $F\Delta t = m(v-u)$

$\Rightarrow F = \dfrac{m(v-u)}{t} = \dfrac{w(0-\sqrt{2gh})}{g\times 0.15} = 5.21$ W

32. COLM $\Rightarrow m_1u_1 + m_2u_2 = (m_1 + m_2)v$

$5 \quad 10^3 \quad 1.2 + 0 = (5+1) \quad 10^3 \quad v \Rightarrow v = 1$ m/s

33. $v_1 = \dfrac{(m_1 - m_2)}{(m_1+m_2)}u_1 + \left(\dfrac{2m_2}{m_1+m_2}\right)u_2$

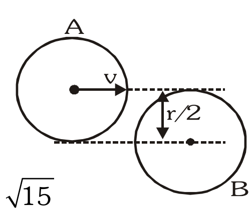
$v_2 = \dfrac{2m_1}{(m_1+m_2)}u_1 + \left(\dfrac{m_2 - m_1}{m_1+m_2}\right)u_2$

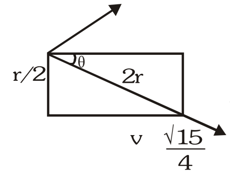
For C : $v_C = \dfrac{2mu}{3m} = \dfrac{2}{3}u$

34. From COLM $Mv_{x_A} + 2Mv_{x_B} = 0 \Rightarrow v_{x_A} = -2v_{x_B}$

so $\vec{v}_A = -2v_x\hat{i} + v\hat{k}$

35. $2r\sin\theta = \dfrac{r}{2}$; $\sin\theta = \dfrac{1}{4}$

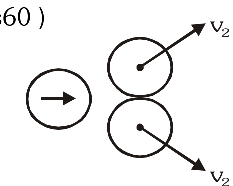




$\cos\theta = \dfrac{\sqrt{15}}{4}$

36. $m \quad 1 = mv_1 + (2mv_2 \quad \cos 60)$

$\Rightarrow v_2 = \dfrac{1}{2}$ & $v_1 = \dfrac{1}{2}$



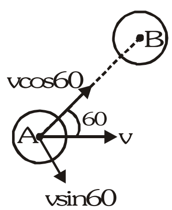
$(KE)_{initial} = \dfrac{1}{2} \quad 1 \quad 1^2 = 0.5$ J

$$(KE)_{final} = \frac{1}{2}(1)\left(\frac{1}{2}\right)^2 + \left\{\frac{1}{2}\times 1 + \times\left(\frac{1}{2}\right)^2\right\}\times 2$$

$= 0.25 + 0.125 = 0.3755$

$\Delta KE = 0.5 - 0.375 = 0.125$ J

37. Component of velocity of A along common normal is v cos 60 and this velocity of A after collision with B is interchanged. Hence A moves along v sin 60 which is normal to common normal.
(उभयनिष्ठ अभिलम्ब के अनुदिश A के वेग का घटक vcos60 है तथा A का यह वेग v के साथ टक्कर के बाद बदल जाता है, अत: A, vsin60 के अनुदिश गति करता है जो कि उभयनिष्ठ अभिलम्ब के लम्बवत् है।)



38. At the time of collision both particles have common velocity and hence the system has minimum kinetic energy. (टक्कर के समय पर दोनों कणों का वेग उभयनिष्ठ होगा तथा निकाय की गतिज ऊर्जा न्यूनतम होगी।)

COME : $mu + 0 = 3mv \Rightarrow v = u/3$

$KE_{initial} = \frac{1}{2}mu^2 = 3J$

$KE_{collision} = \frac{1}{2}(3m)v^2 = \frac{1}{2}(3m)\frac{u^2}{9} = 1J$

$PE_{collision} = (3-1) = 2J$

Total energy remains constant and hence KE of system First decreases & then increases.
(कुल ऊर्जा नियत रहती है अत: निकाय की गतिज ऊर्जा पहले घटती है तथा फिर बढ़ती है।)

39. At the time of maximum compression,
(अधिकतम संपीड़न के समय पर)

COLM : $mu = 2mv$ (for A & B) $\Rightarrow v = u/2$

COME : $\frac{1}{2}mu^2 = \frac{1}{2}2mv^2 + \frac{1}{2}kx^2 \Rightarrow x = v\sqrt{\frac{m}{2k}}$

40. PE of solid sphere (ठोस गोले की स्थितिज ऊर्जा)

$= mgR = mg\frac{D}{2} = \rho g D^4\left(\frac{\pi}{12}\right)$

PE of solid cube (ठोस घन की स्थितिज ऊर्जा)

$= mg\frac{D}{2} = \rho g D^4\left(\frac{1}{2}\right)$

PE of solid cone (ठोस शंकु की स्थितिज ऊर्जा)

$= mg\frac{D}{4} = \rho g D^4\left(\frac{\pi}{48}\right)$

PE of solid cylinder (ठोस बेलन की स्थितिज ऊर्जा)

$= mg\frac{D}{2} = \rho g D^4\left(\frac{\pi}{8}\right)$

41. COLM : $mu + 0 = (m + M)v \Rightarrow v = \left(\frac{m}{M+m}\right)u$

KE after collision (टक्कर के बाद गतिज ऊर्जा):

$$= \frac{1}{2}(m+M)\times\left(\frac{m}{m+M}\right)^2 u^2 = \frac{m^2u^2}{2(m+M)}$$

42. $p_i = -mv$, $p_f = m(v + 2u)$ $\therefore$ $\Delta p = 2m(v + u)$

$\therefore$ Force (बल)$= \frac{\Delta p}{\Delta t} = \frac{2m(v+u)}{\Delta t}$

$KE_{initial} = \frac{1}{2}mu^2$, $KE_{final} = \frac{1}{2}m(2v+u)^2$

$\therefore\ \Delta KE = \frac{1}{2}m[4v^2 + u^2 + 4uv - u^2]$

$= \frac{1}{2}m[4v(v+u)] = 2mv(u+v)$

EXERCISE –III

Fill in the blank

1. By applying conservation of momentum
(संवेग संरक्षण के द्वारा)

$$0 = m\vec{v}_1 + m\vec{v}_2 + 2m\vec{v}_3$$

$$v_3 = \frac{|\vec{v}_1 + \vec{v}_2|}{2} = \frac{\sqrt{2}v}{2} = \frac{v}{\sqrt{2}}$$

Total energy released in explosion
(विस्फोट में मुक्त कुल ऊर्जा)

$$\frac{1}{2}mv^2 + \frac{1}{2}mv^2 + \frac{1}{2}\times 2m\left(\frac{v}{\sqrt{2}}\right)^2 = \frac{3mv^2}{2}$$

2. Area under the F-t curve = impulse=5 10^{-3} N-s
(F-t वक्र के अन्तर्गत क्षेत्रफल=आवेग)

3. $h = (e^2)^n\, h_0 \Rightarrow h = (0.8^2)^3\, h_0 \Rightarrow h = (0.8)^6\, h_0$

6. Momentum is conserved in all collision.
(सभी टक्करों में संवेग संरक्षित रहता है)

7. COLM : $mv = (m + A\rho x)v' \Rightarrow v' = \frac{mv}{m + A\rho x}$

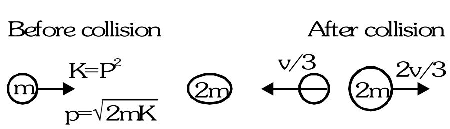
8. Loss in KE= $\frac{m_1 m_2}{2(m_1 + m_2)}(1-e^2)(u_1 - u_2)^2$

Here $e = \frac{5}{15} = \frac{1}{3}$

$$\therefore \Delta KE = \frac{(3)(2)}{2(3+2)}\left(1 - \frac{1}{9}\right)(15)^2 = 120J$$

Match the Column

1. By applying conservation of momentum
(संवेग संरक्षण के द्वारा)



$m_1u_1 + 2m(0) = mv_1 + 2mv_2$ ....(i)

Also $u = v_2 - v_1$ ....(ii)

$$v_2 = \frac{2v}{3} \text{ and } v_1 = -\frac{v}{3};\ p_2 = \frac{4mv}{3} = \frac{4p}{3},$$

$$p_1 = -\frac{mv}{3} = -\frac{p}{3};\ K_2 = \frac{8K}{9};\ K_1 = \frac{K}{9}$$

2. Impulse = change in momentum
(आवेग–संवेग में परिवर्तन)

(A) For body M : $p = |\vec{p}_f - \vec{p}_\ell| \Rightarrow p_f = p$

(B) For body 2M : $p = |\vec{p}_f - \vec{p}_\ell| \Rightarrow p_f = 2p$

(C) $e = \frac{v_2 - v_1}{u_1 - u_2} \Rightarrow \frac{\frac{p}{M} - \frac{p}{M}}{\frac{2p}{M} - \frac{p}{2M}} = 0 \Rightarrow e = 0$

3. For 1kg $v_1 = (2t)\hat{i} = 4\hat{i}$; $a_i = 2\hat{i} = 2\hat{i}$
For 2kg $v_2 = t^2\hat{j} = 4\hat{j}$; $a_2 = 2t\hat{j} = 4\hat{j}$

(A) Acceleration of centre of mass = $\frac{m_1a_1 + m_2a_2}{m_1 + m_2}$
(द्रव्यमान केन्द्र का त्वरण)

$$\vec{a}_{cm} = \frac{2}{3}\hat{i} + \frac{8}{3}\hat{j} \Rightarrow a_{cm} = \sqrt{\frac{4}{9} + \frac{64}{9}} \Rightarrow \frac{\sqrt{68}}{3}\ m/s^2$$

$$f = ma_{cm} = \sqrt{68}\ N$$

(B) Velocity of centre of mass (द्रव्यमान केन्द्र का वेग)

$$\vec{v}_{cm} = \frac{m_1v_1 + m_2v_2}{m_1 + m_2} = \left(\frac{4}{3}\hat{i} + \frac{8}{3}\hat{j}\right) m/s$$

$$\Rightarrow |\vec{v}_{cm}| = \sqrt{\frac{16}{9} + \frac{64}{9}} \Rightarrow \frac{\sqrt{80}}{3}$$

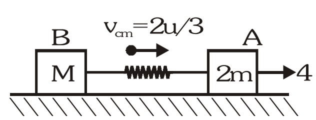
(C) $\vec{v}_{cm} = \frac{1(2t)\tilde{i} + 2(t^2\tilde{j})}{1+2} = \frac{2}{3}t\tilde{i} + \frac{2}{3}t^2\tilde{j}$

Displacement =

$$\int_0^2 \vec{v}_{cm}dt = \left[\frac{3}{2}\left(\frac{t^2}{2}\right)^2 \tilde{i} + \frac{2}{3}\left(\frac{t^3}{3}\right)\tilde{j}\right]_0^2 = \frac{4}{3}\tilde{i} + \frac{16}{9}\tilde{j}$$

$$\Rightarrow |\text{Dispalcement}| = \sqrt{\left(\frac{4}{3}\right)^2 + \left(\frac{16}{9}\right)^2} = \frac{20}{9}\text{units}$$

4. As no external force acts on the system velocity of centre of mass remain same (चूंकि निकाय पर कोई बाह्य बल आरोपित नहीं है अत: द्रव्यमान केन्द्र का वेग समान बना रहता है)



$$v_{cm} = \frac{2m(u)}{3m} = \frac{2u}{3}$$

In frame of centre of mass velocity of B is 2u/3 and It oscillates from $\left(-\frac{2u}{3}, \frac{2u}{3}\right)$

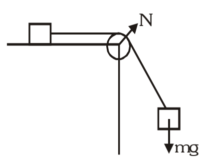
In frame of centre of mass velocity of A is u/3 and It oscillates from $\left(-\frac{u}{3}, \frac{u}{3}\right)$. In ground frame velocity of B $\left[0, \frac{4u}{3}\right]$. In ground frame velocity of A $\left[\frac{u}{3}, u\right]$

and by conservation of energy $\Delta K.E. = \Delta U_S$

(द्रव्यमान केन्द्र निर्देश तंत्र में B का वेग 2u/3 तथा यह $\left(-\frac{2u}{3}, \frac{2u}{3}\right)$ में दोलन करेगा। द्रव्यमान केन्द्र निर्देश तंत्र में A का वेग u/3 तथा यह $\left(-\frac{u}{3}, \frac{u}{3}\right)$ से दोलन करेगा। जमीन तंत्र में B का वेग $\left[0, \frac{4u}{3}\right]$ तथा जमीन तंत्र में A का वेग $\left[\frac{u}{3}, u\right]$ तथा ऊर्जा संरक्षण द्वारा $\Delta K.E. = \Delta U_S$)

5. (A) Net force on block m acts in downward direction
(ब्लॉक m पर कुल बल नीचे की दिशा में लगता है)
$\therefore$ Acceleration of centre of mass is in downward direction. ($\therefore$ द्रव्यमान केन्द्र का त्वरण नीचे की दिशा में है)

(B) Net force acts in downward as well as in horizontal direction. (कुल बल क्षैतिज दिशा में तथा नीचे की ओर लगता है)



$\therefore$ a_{cm} moves both in horizontal & vertical direction.
($\therefore$ a_{cm} दोनों ओर तथा ऊर्ध्वाधर दिशा में गतिशील होता है)

(C) As the mass of monkey & block is same both moves upward.
(चूंकि बन्दर का द्रव्यमान तथा ब्लॉक का द्रव्यमान समान है अत: दोनों ऊपर की ओर गति करते है।)
$\therefore$ Centre of mass moves upward
($\therefore$ द्रव्यमान केन्द्र ऊपर की ओर गतिशील होगा)

(D) Centre of mass of the system does not moves
(निकाय का द्रव्यमान केन्द्र गति नहीं करता है)

Comprehension # 1

1. By applying conservation of momentum
(संवेग संरक्षण द्वारा)
$mv_1 + Mv_2 = 0 \Rightarrow v_1 = -4v_2$ ...(i)
By applying conservation of energy
(ऊर्जा संरक्षण द्वारा)

$\frac{1}{2}mv_1^2 + \frac{1}{2}Mv_2^2 = mgh \Rightarrow \frac{v_1^2}{2} + 2v_2^2 = 20$ ...(ii)

$v_2 = \sqrt{2} m/s$; $v_1 = 4\sqrt{2}$ m/s

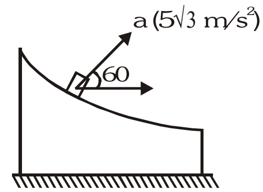
2. When 'm' leaves the wedge 'M' then wedge moves distance 'x' in left side (जब वेज M से m को छोड़ा जाता है तो वेज से बांयी ओर x दूरी चलता है)
$\therefore$ m (4–x) = Mx $\Rightarrow$ x = 0.8 m
$\therefore$ Co-ordinate where block will leave wedge
(निर्देशांक, जहां ब्लॉक वेज को छोड़ता है)
x = 4– 0.8 = 3.2

Time for m will strike the ground is = $\sqrt{\frac{2\times 2}{10}}$

(m के लिए वह समय जब यह जमीन के टकरायेगा)

$\therefore x_f = 3.2 + 4\sqrt{2} \quad \frac{2}{\sqrt{10}} = 6.8$ m

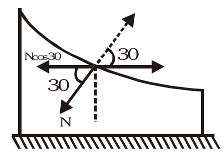
3. $a_{cm} = \frac{m_1a_1 + m_2a_2}{m_1 + m_2}$



(1) a cos 60 = 4a

$a_M = \frac{5\sqrt{3}}{8}$ m/s²

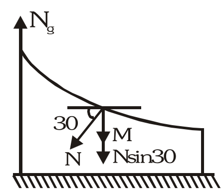
N cos 30 = 4ma$_M$



$N\frac{\sqrt{3}}{2} = 4(1)\frac{5\sqrt{3}}{8}$

$\Rightarrow$ N = 5 Newton

4. $N_g = N \sin 30 + 40 \Rightarrow N_g = 42.5$



Comprehension # 2

By applying conservation of momentum
(संवेग संरक्षण द्वारा)
$2(6) + 1(4) = 1v_2 + 2v_1$; $16 = v_2 + 2v_1$ ...(i)
By applying newton law of collision
(न्यूटन के टक्कर के नियम द्वारा)

$1 = \frac{v_2 - v_1}{2} \Rightarrow v_2 - v_1 = 2$ ...(ii)

$\Rightarrow v_2 = \frac{20}{3}$, $v_1 = \frac{14}{3}$

1. Impulse = change in momentum $= \frac{20}{3} - 4 = \frac{8}{3}$ N–s
(आवेग = संवेग में परिवर्तन)

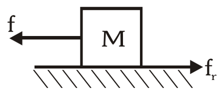
2. To change the direction of a block impulse should be greater than 12 N–s (ब्लॉक अपनी दिशा परिवर्तित कर लेगा जब आवेग 12 N-s से अधिक हो)

Comprehension # 3

As horizontal velocity (i.e. velocity along the surface) remains constant so required time = $\frac{30}{5} = 6s$

(चूंकि क्षैतिज वेग (अर्थात् सतह के अनुदिश वेग) नियत रहता है, अतः अभिष्ट समय $= \frac{30}{5} = 6s$)

Comprehension # 4



1. $f = v\frac{dm}{dt} = 2(20) = 40 \Rightarrow f_r = \mu mg$

   $m = 40 = M_0 - 2(t) \Rightarrow t = 5$ sec

   so $(0.1)(50-2t) = 40 \Rightarrow t = 5$ sec

2. $v = v\,\ell n\left(\frac{m}{m_0}\right) - gt = 20\,\ell n\left[\frac{4}{3}\right] - gt = 20\,(0.28) - 5$

   $= 5.6 - 5 = 0.6 m/s$

Comprehension # 5

1. $\vec{r}_{cm} = \frac{m_1\vec{r}_{10} + m_2\vec{r}_{20}}{m_1 + m_2} = \frac{1(3\hat{i}) + 2(9\hat{j})}{1+2} = (\hat{i} + 6\hat{j})m$

   $\vec{v}_{cm} = \frac{m_1\vec{v}_1 + m_2\vec{v}_2}{m_1 + m_2} = \frac{1(3\hat{i}) + 2(6\hat{j})}{1+2}(\hat{i} + 4\hat{j})m/s$

   Now $\Delta\vec{r}_{cm} = \vec{v}_{cm}t \Rightarrow \vec{r}_{cm} - \vec{r}_{cm_0} = \vec{v}_{cm}t$

   $\Rightarrow (x-1)\hat{i} + (y-6)\hat{j} = (\hat{i} + 4\hat{j})t$

   $\Rightarrow x = 1 + t$ and $y = 6 + 4t \Rightarrow y = 4x + 2$

2. $\vec{a}_{cm} = \frac{m_1\vec{a}_1 + m_2\vec{a}_2}{m_1 + m_2} = \frac{1(-2\hat{i}) + 2(-2\hat{j})}{1+2}$

   $= -\frac{2}{3}(\hat{i} + 2\hat{j})m/s^2$

   By using $\vec{v}_{cm} = \vec{u}_{cm} + \vec{a}_{cm}t$; we get t =3s

3. By using $s = ut + \frac{1}{2}at^2$ for individual partiles

   For I[st] particle: $s_x = (3)(1.5) + \frac{1}{2}(2)(1.5)^2 = 2.25$ m

   For II[nd] particle $= s_y = (6)(3) + \frac{1}{2}(-2)(3)^2 = 9m$

   Therefore $x_{cm} = \frac{2.25}{3} + 1 = 1.75$ m and

   $y_{cm} = \frac{2(9)}{3} + 6 = 12m$

Comprehension # 6

1. As no external force acts on the two blocks friction acts like an internal forces and the two blocks will move with common velocity. (चूंकि दोनों ब्लॉकों पर कोई बाह्य बल कार्य नहीं करता है इसलिए घर्षण बल एक आन्तरिक बल है तथा दोनों ब्लॉक उभयनिष्ठ वेग से गति करते हैं।)

   By applying conservation of momentum

   (संवेग संरक्षण नियम द्वारा)

   $(1kg)(15\ m/s) = 1v = 2v \Rightarrow v = 5\ m/s$

   $P_1 = 5$ N–s ; $P_2 = 10$ N–s

2. $\frac{dp}{dt} = f_{ext}$

   For block of 1 kg

   friction $f_r = \mu mg = 0.4\quad 1\quad 10 = 4$ N

3. $v = u + at \Rightarrow 5 = 15 - (4)t \Rightarrow t = 2.5$ sec

Comprehension # 7

1. As $ma_{cm} = f \Rightarrow a_{cm} = \frac{f}{2m}$ so $s_{cm} = \frac{1}{2}at^2 = \left(\frac{f}{4m}\right)t^2$

2. $x_{cm} = \frac{m_1x_1 + m_2x_2}{2m} \Rightarrow x_1 + x_2 = \frac{f}{2m}t^2$

   $x_1 - x_2 = x_0$

   Therefore $x_1 = \frac{f}{4m}t^2 + \frac{x_0}{2}$

   From above equations $x_2 = \frac{ft^2}{4m} - \frac{x_0}{2}$

Comprehension # 8

1. By COLM (संवेग संरक्षण नियम द्वारा)

   $0 = m_A v_A + m_B v_B$; $\vec{v}_B = -\frac{m_A}{m_B}\vec{v}_A$

   Both velocity are opposite in direction

   (दोनों वेग दिशा में विपरीत होंगे)

   $\therefore$ II, IV, V

2. If $m_A = m_B \Rightarrow \vec{v}_B = -\vec{v}_A \Rightarrow$ Graph II

   ; $m_A > m_B \Rightarrow v_B > v_A \Rightarrow \tan\theta_B > \tan\theta_A \Rightarrow$ Graph IV

   If $m_A < m_B \Rightarrow v_A > v_B \Rightarrow$ Graph IV

3. v_{cm} is not zero in graph (I), (III) and (VI)

   (वक्र (I), (III) एवं (VI) में v_{cm} का मान शून्य नहीं होगा)

EXERCISE –IV(A)

1. (i) The centre of mass remains at O as the excluded masses are symmetrically placed.
(द्रव्यमान केन्द्र O पर होगा जो कि बचे हुए द्रव्यमान केन्द्र की सममिती पर होगा)
(ii) CM shifts from 0 to 3 diagonally
(द्रव्यमान केन्द्र 0 से 3 विकर्णत दूरी पर होगा)
(iii) CM shifts along OY
(द्रव्यमान केन्द्र OY की ओर होगा)
(iv) CM does not shift.
(द्रव्यमान केन्द्र विस्थापित नहीं होगा)
(v) CM shifts diagonally from 0 to 4.
(द्रव्यमान केन्द्र 0 से 4 विकर्णत दूरी पर होगा)
(vi) CM doesnot shift.
(द्रव्यमान केन्द्र विस्थापित नहीं होगा)

2. $$x_{cm} = \frac{(m\times 0)+(2m\times a)+(3m\times a)+(4m\times 0)}{m+2m+3m+4m} = \frac{a}{2}$$
$$y_{cm} = \frac{(m\times 0)+(2m\times 0)+(3m\times a)+(4m\times a)}{m+2m+3m+4m} = \frac{7a}{10}$$

3. Length of rod = $\sqrt{(4-2)^2+(2-5)^2} = \sqrt{13}\,m$
$$x_{cm} = \frac{(3\times 2)+(2\times 4)}{3+2} = \frac{14}{5};\ y_{cm} = \frac{(3\times 5)+(2\times 2)}{3+2} = \frac{19}{5}$$

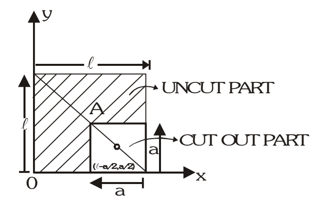
4. $$x_{cm} = \frac{\left(M\times\frac{a}{2}\right)+(M\times 0)+\left(M\times\frac{a}{2}\right)}{M+M+M} = \frac{a}{3}$$
$$y_{cm} = \frac{(M\times 0)+\left(M\times\frac{a}{2}\right)+\left(M\times\frac{a}{2}\right)}{M+M+M} = \frac{a}{3}$$

5. $$y_{cm} = \frac{\left\{\rho\frac{\pi(2R)^2}{3}\times 4R\right\}\times R+\left\{12\rho\times\frac{4}{3}\pi R^3\right\}\times 5R}{\left\{\rho\frac{\pi(2R)^2}{3}\times 4R\right\}+\left\{12\rho\times\frac{4}{3}\pi 4R^3\right\}} = 4R$$

6. $$x_{cm} = \frac{\left(\rho 2r^2\right)\frac{r}{2}-\left(\rho\frac{\pi r^2}{2}\right)\left(\frac{4r}{3\pi}\right)}{\left(\rho 2r^2\right)-\left(\rho\frac{\pi r^2}{2}\right)} = \frac{2r}{3(4-\pi)}$$

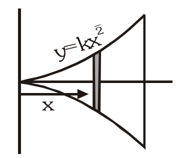
7. $$x_{cm} = \frac{\left(\rho\frac{\pi\times 56^2}{4}\right)\times 28+\left(\rho\frac{\pi\times 42^2}{4}\right)\times 35}{\left(\rho\frac{\pi\times 56^2}{4}\right)-\left(\rho\frac{\pi\times 42^2}{4}\right)}$$
=9cm from left edge

8. $$\vec{y} = \frac{(\rho\ell^2)\times\frac{\ell}{2}-(\rho a^2)\frac{a}{2}}{\rho\ell^2-\rho a^2} = a \Rightarrow \ell^2 - a\ell - a^2 = 0$$



$$\Rightarrow a = \frac{(\sqrt{5}-1)\ell}{2}\,x$$

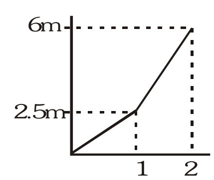
9. $$x_{cm} = \frac{\int x\,dm}{\int dm} = \frac{\int_0^a x\rho y\,dx}{\int_0^a \rho y\,dx} = \frac{3}{4}a$$



10. (i) CM does not shift
(द्रव्यमान केन्द्र विस्थापित नहीं होता है)
(ii) Plank moves towards right.
(तख्ता दांयी ओर गति करता है)
(iii) $$\Delta x_{CM} = \frac{m_1\Delta x_1 + m_2\Delta x_2 + M\Delta x}{m_1+m_2+M} = 0$$
$$\Rightarrow 0 = \frac{50(x+2)+70(x-2)+80x}{50+70+80}$$
$\Rightarrow x = 0.2m$ (right)
(iv) $\Delta x_{m_1} = x + 2 = 2.2$ m (right)
(v) $\Delta x_{m_2} = x - 2 = -1.8m$ (left)

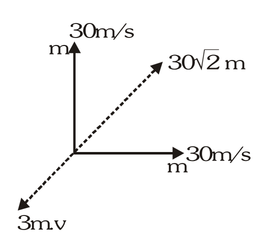
11. For $0 \le t < 1$ from $v_{cm} = \dfrac{m_1v_1+m_2v_2}{m_1+m_2}$

$$\Rightarrow 2 = \frac{(1)(1)+(2)v_2}{3}$$
$\Rightarrow v_2 = 2.5\ ms^{-1}$
$\Rightarrow x_2 = 2.5t$



for $1 \le t < 2$, $2 = \dfrac{(1)(-1)+2(v_2)}{3}$
$\Rightarrow v_2 = 3.5ms^{-1}$
$\Rightarrow x_2 = 2.5 + 3.5\ (t-1)$

12. COLM $\Rightarrow 3mv = 30\sqrt{2}m \Rightarrow v = 10\sqrt{2}\ m/s$



13. Velocity of mass-1 when string is in normal length. (द्रव्यमान 1 का वेग जब रस्सी मूल लम्बाई में हो)

$$v_A = \sqrt{6g\ell - 2g\ell} = 2\sqrt{g\ell}$$

Now impulsive tension acts on both bodies to come to common velocity (दोनों वस्तुओं पर आरोपित आवेगीय तनाव के कारण उभयनिष्ठ वेग) $v_{common} = \frac{v_A}{2} = \sqrt{g\ell}$

Displacement of C.M. when string becomes taut. (द्रव्यमान केन्द्र का विस्थापन जब रस्सी तनी हुई हो)

$$\Delta y_{1cm} = \frac{m \times \ell + m \times 0}{m + m} = \frac{\ell}{2}$$

Displacment of CM when masses reach the max. height (द्रव्यमान केन्द्र का विस्थापन जब द्रव्यमान अधिकतम ऊंचाई पर हो)

$$\Delta y_{2m} = \frac{v^2}{2g} = \frac{\ell}{2} \therefore \Delta y_{cm} = \frac{\ell}{2} + \frac{\ell}{2} = \ell$$

14. From work energy theorem (कार्य–ऊर्जा प्रमेय से)

$W_F + W_g = \Delta KE$

$\Rightarrow F_{avg}(h_2 - h_1) - mg(h_3 - h_1) = 0 \Rightarrow F_{avg} = \frac{mg(h_3 - h_1)}{(h_2 - h_1)}$

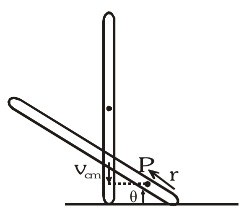
15. In the presence of gravity, the CM shifts along vertically downward direction.

(गुरूत्व की उपस्थिति में द्रव्यमान केन्द्र ऊर्ध्वाधर नीचे की दिशा के अनुदिश विस्थापित होता है)

For point P : $y = r\sin\theta$

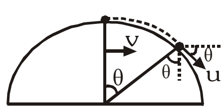
$x = (\ell/2 - r)\cos\theta$

$$1 = \left(\frac{x}{\frac{\ell}{2} - r}\right)^2 + \left(\frac{y}{r}\right)^2$$



16. Let v = velocity of wedge and u = veloicty of particle relative to wedge

(माना v = वेज का वेग तथा u= वेज के सापेक्ष कण का वेग)



COLM

$\Rightarrow m(v + u\cos\theta) + 4mv = 0$

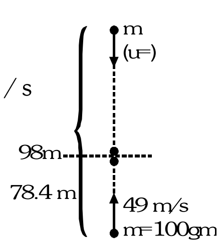
$\Rightarrow u\cos\theta = -5v;\ \omega = \frac{u}{R} = \frac{5v}{R\cos\theta}$

17. $\left(\frac{1}{2}gt^2\right) + \left(49t - \frac{1}{2}gt^2\right) = 98$

$\Rightarrow t = 2s\ \ \vec{u}_1 = gt(\hat{j}) = -19.6\hat{j}\,m/s$

$\vec{u}_2 = (u - gt)\hat{j} = +29.4\hat{j}\ m/s$

$v_f = \frac{\vec{u}_1 + \vec{u}_2}{2} = 4.9\,m/s$



$h = 4.9 \quad 2 - \frac{1}{2} \times 9.8 \times 2 \times 2 = 78.4m$

For the combined mass (संयुक्त द्रव्यमान के लिए)

$$x = ut + \frac{1}{2}\ 9.8\,t^2,\ t = 4.53$$

$\therefore$ Total time of height (ऊंचाई का कुल समय)

$= 2 + 4.53 = 6.53$ sec.

18. $\Delta x_{cm} = \frac{M\Delta x_1 + m\Delta x_2}{M + m} \Rightarrow 0 = \frac{Mx + m(x + R - r)}{M + m}$

Distance moved by the cylinder $x = -\frac{m(R - r)}{M + m}$

(बेलन द्वारा तय दूरी)

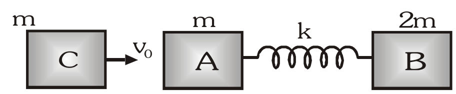
For motion along x-axis $0 = m(v_1 + v_2) + mv_2$....(i)

(x अक्ष के अनुदिश गति के लिए)

$mg(R - r) = \frac{1}{2}m\,(v_1 + v_2)^2 + \frac{1}{2}Mv_2^2$ ...(ii)

$$\therefore\ v_2 = m\sqrt{\frac{2g(R - r)}{M(M + m)}}$$

19.



After collision (टक्कर के बाद): $u_A = v_0$

(i) When $(v_{inst})_A = (v_{inst})_B \Rightarrow mv_0 = 3mv \Rightarrow v = v_0/3$

(ii) COME

$\Rightarrow \frac{1}{2}mv_0^2 = \frac{1}{2}(3m)v^2 + \frac{1}{2}kx_0^2$

$\Rightarrow k = \frac{2m}{3}\left(\frac{v_0}{x_0}\right)^2$

20. $\overset{u}{\rightarrow}$ o o = $\overset{v}{\rightarrow}$ oo

m_1 m_2 $(m_1 + m_2)$

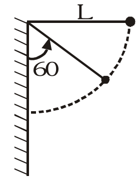
COLM $\Rightarrow m_1u + 0 = (m_1 + m_2)v$ ....(i)

Energy equation (ऊर्जा का समीकरण)

$\left(\frac{1}{2}mu^2\right) \times \frac{2}{3} = \frac{1}{2}(m_1 + m_2)v^2$ ..(ii)

$\Rightarrow \frac{m_1}{m_2} = 2:1$

21.



After I[st] collision

(प्रथम टक्कर के बाद)

$\frac{2}{\sqrt{5}}\sqrt{2g\ell} = \sqrt{2g\ell(1 - \cos\theta)}$

$\cos\theta = \frac{1}{5}$

For II[nd] collision (द्वितीय टक्कर के लिए)

$$\frac{2}{\sqrt{5}}\cdot\frac{2}{\sqrt{5}}\sqrt{2g\ell} = \sqrt{2g\ell(1-\cos\theta)}$$

For n^{th} collision (nवीं टक्कर के लिए)

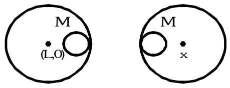
$$\left(\frac{2}{\sqrt{5}}\right)^n = \sqrt{1-\cos\theta} \Rightarrow \left(\frac{4}{5}\right)^n = 1-\cos\theta$$

$$\cos\theta = 1-\left(\frac{4}{5}\right)^n$$

Put n=0,1,2,3 and get answer

22. $x_{cm} = \dfrac{4ML + M(L+5R)}{5M} = \dfrac{4MX + M(X-5R)}{5M}$

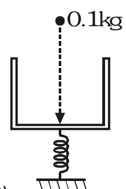
$\Rightarrow$ x = L + 2R



23. In this elastic collision velocity of masses are exchanged. (प्रत्यास्थ टक्कर में द्रव्यमानों के वेग आपस में बदल जाते हैं)

So $v_A = 0 \Rightarrow$ A does not rise

24. COME $\Rightarrow mg(h + x_0) = \dfrac{1}{2}kx_0^2$



$\Rightarrow mg(0.24 + 0.01)$

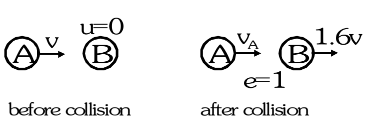
$= \dfrac{1}{2}k(0.01\ \ \ 0.01)$ ....(i)

& $mg(h + 0.04) = \dfrac{1}{2}k(0.04)^2$ ...(ii)

Equation (i) divided by (ii) h= 3.96 m

25. No, KE is not conserved during the short time of collision. (नहीं, अल्पावधि में गेंदो की टक्कर के दौरान गतिज ऊर्जा संरक्षित नहीं रहती है)

26.



$$\text{COLM} = -\frac{(1.6v)-v_A}{0-v}\ v_A = 0.6V \quad ...(i)$$

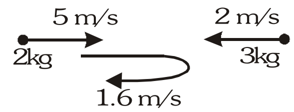
COLM $\Rightarrow m_Av = m_A\ (0.6V) + m_B(1.6\ v)$

$$\frac{m_A}{m_B} = 4 \qquad ...(ii)$$

$$\Delta K_B = \frac{1}{2}m_B(1.6\ v)^2 = 0 = 2m_B\ (0.8\ v)^2$$

$$K_A = \frac{1}{2}(4m_B)v^2 = 2m_Bv^2 \Rightarrow \frac{\Delta K_B}{K_A} = 0.64 = 64\%$$

27.



(A) $v_{cm} = \left[\dfrac{(2\times5)-(3\times2)}{2+3}\right]\hat{i} = \dfrac{4}{5}\hat{i}\ m/s$

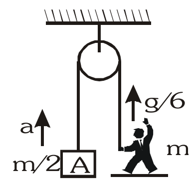
(B) COLM $\Rightarrow (2\ \ \ 5) + 3(-2) = +2(-1.6) + 3v_2$

$\Rightarrow v_2 = 2.4$ m/s

(C) $e = -\left(\dfrac{v_2-v_1}{u_2-u_1}\right) = \dfrac{4}{7}$

28. COLM: implies that $\vec{v}_C$ & $\vec{v}_B$ are opposite to each other. (संवेग संरक्षण से $\vec{v}_C$ व $\vec{v}_B$ एक-दूसरे से विपरीत होंगे)

29. $mg - T = m\left(a - \dfrac{g}{6}\right)$...(i)



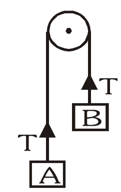
$\Rightarrow T - \dfrac{m}{2}g = \dfrac{m}{2}a$ ...(ii)

For solving eq. (i) & (ii)

$a = \dfrac{4g}{9}$ & $T = \dfrac{13}{18}mg$

30. Velocity of B when string is in natural length (B का वेग, जब रस्सी पूर्ण लम्बाई में हो)

$= \quad u_B = \sqrt{2gh} = \sqrt{2g}\ (h=1)$



Impulse equation (आवेग समीकरण)

$\Rightarrow -T\Delta t = m\ [v - \sqrt{2g}\,]$ ...(i)

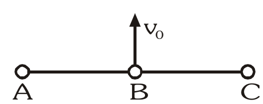
$T.\Delta t = m[v-0]$ ...(ii)

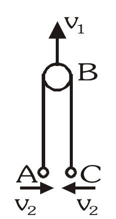
On solving eq. (i) & (ii) : $v = \dfrac{\sqrt{2g}}{2} = \dfrac{g}{\sqrt{2}}$

$\therefore$ Distance travelled by A before coming to rest, (विरामावस्था में आने से पहले A द्वारा तय की गई दूरी)

$$s = 1 + \frac{v^2}{2g} = 1.25m$$

31.





COLM $\Rightarrow mv_0 = 3mv_1$

$v_1 = \dfrac{v_0}{3}$ ...(i)

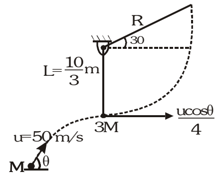
COME

$$\Rightarrow \frac{1}{2}mv_0^2 = \frac{1}{2}(3m)v_1^2 + \frac{1}{2}(2m)v_0^2 \Rightarrow v_2 = \frac{v_0}{\sqrt{3}}$$

Therefore velocity of A = $\sqrt{v_1^2+v_2^2} = 6$ m/s

EXERCISE –IV(B)

1. (i) $\left(\dfrac{u\cos\theta}{4}\right)^2 = 2g\left(\dfrac{3R}{2}\right)$

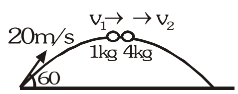


$\Rightarrow$ cos θ = 4/5 $\Rightarrow$ θ = 37

(ii) $x = \dfrac{R}{2} = \dfrac{u^2\sin^2\theta}{2g} = 120\text{m}$

$y = H = \dfrac{u^2\sin^2\theta}{2g} = 45\text{m}$

2. COLM :1 v_1 + 4 v_2 = 5 20 cos 60 = 50 ...(i)



COME $\Rightarrow$ $\dfrac{1}{2}$ 1 v_1^2 + $\dfrac{1}{2}$ 4 v_2^2

$= \left[\dfrac{1}{2}5\times(10)^2\right]\times 2$ ...(ii)

$\Rightarrow$ v_1 = –10 or 30 m/s & v_2 = 15 or 5 m/s

$\Rightarrow$ Δv = 25 m/s

Time to fall down to ground

(जमीन पर गिरने में लगा समय) $= \dfrac{\sqrt{2h}}{g} = \dfrac{\sqrt{3}}{.98}$ sec.

∴ Separation between particles (कणों के मध्य दूरी) = Δv.t = 44.2m

3. $\Delta x_{cm} = 0 = \dfrac{mx_0 + m_1(x_0 - h\cot\alpha) + m_2(x_0 - h)}{m + m_1 + m_2}$

$\Rightarrow x_0 = \dfrac{h(m_2 + m_1\cot\alpha)}{(m + m_1 + m_2)}$

4. $u_s = \sqrt{2g(1-\cos 60°)} = \sqrt{2g\times\dfrac{1}{2}} = 3.13\,\text{m/s}$

COLM $\Rightarrow$ $5u_B$ = 4 $u_s \Rightarrow u_B$ = $\dfrac{4}{5}\sqrt{g}$ = 2.53 m/s

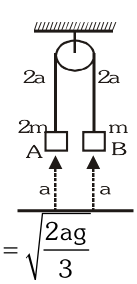
Energy equation (ऊर्जा समीकरण)

$\dfrac{1}{2}m\left(\dfrac{4}{5}\sqrt{g}\right)^2$ = μmg 0.8 $\Rightarrow$μ = 0.4

$e = -\left(\dfrac{v_2 - v_1}{u_2 - u_1}\right) = 0.8$

5. 2mg – T = 2m.a ...(i)

T – mg = m.a ...(ii)



On solving eq. (i) & (ii) $a = \dfrac{g}{3}$

(a) $v_B = \sqrt{u^2 + 2as} = \sqrt{0 + 2\left(\dfrac{g}{3}\right)a} = \sqrt{\dfrac{2ag}{3}}$

(b) $s = ut + \dfrac{1}{2}at^2$, $a = 0 + \dfrac{1}{2}\left(\dfrac{g}{3}\right)t^2$, $t = \sqrt{\dfrac{6a}{g}} = \dfrac{3v}{g}$

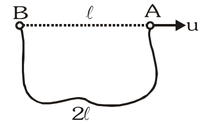
(c) $t = \dfrac{2v}{g}$

6. (a) mu – TΔt = mv ...(i)

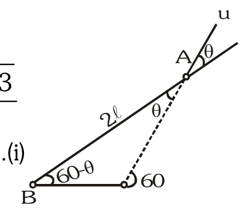
TΔt = mv ...(ii)

On solving eq. (i) & (ii)

v = u/2



(b) $\dfrac{2\ell}{\sin 120°} = \dfrac{\ell}{\sin\theta}$



$\sin\theta = \dfrac{\sqrt{3}}{4}$; $\cos\theta = \dfrac{\sqrt{13}}{4}$

mu cos θ – TΔt = mv ...(i)

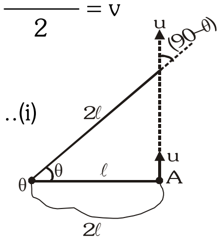
$\Rightarrow$TΔt = mv..(ii)

On solving eq. (i) & (ii) $\dfrac{u\cos\theta}{2} = v$

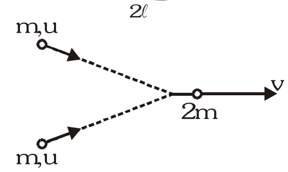
(c) 2ℓ cos θ = ℓ $\Rightarrow$ θ = 60

mucos 30 – TΔt = mv ..(i)

$T\Delta t = \dfrac{mu\sqrt{3}}{4}$ ...(ii)



7. $2mu\cos\dfrac{\theta}{2} = 2mv$



$\Rightarrow v = u\cos\dfrac{\theta}{2} = \dfrac{u}{2}$

$\Rightarrow \cos\dfrac{\theta}{2} = \dfrac{1}{2}$; $\dfrac{\theta}{2} = 60°$, θ = 120

8. COLM $\Rightarrow$ mu = (m + ρAx)v $\Rightarrow v = \dfrac{mu}{m + \rho Ax}$

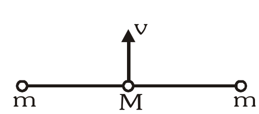
$\Rightarrow \dfrac{dx}{dt} = \dfrac{mu}{m + \rho Ax}$ $\Rightarrow \int_0^{150}(m + \rho Ax)dx = \int_0^{150} mudt$

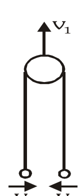
$\Rightarrow \left(mx + \rho\dfrac{Ax^2}{2}\right)$ = mut

$\Rightarrow$ $10^{-2}x$ + 10^{-3} $\dfrac{10^{-4}}{2}x^2$ = 10^{-2} 10^3 150

x = 10^5 m

9.





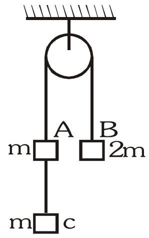
(i) COLM $\Rightarrow$ (m + m) 0 + Mv = (M + 2m)v_1

$v_1 = \left(\dfrac{M}{M + 2m}\right)v$

(ii) COME $\Rightarrow$ $\dfrac{1}{2}MV^2 = \left[\dfrac{1}{2}m(v_1^2 + v_2^2)\right]\times 2 + \dfrac{1}{2}mv_1^2$

Net velocity $v_0 = \sqrt{v_1^2 + v_2^2} = v\dfrac{\sqrt{2M(M+m)}}{(M+2m)}$

10. $2mg - T = 2ma$; $T - mg = ma$; $a = \dfrac{g}{3}$



Velocity of m & 2m after falling through a distance (x दूरी गिरने के बाद m व 2m द्रव्यमान का वेग)

$$x = \sqrt{2ax} = \sqrt{\frac{2gx}{3}}$$

Impulse equation (आवेग समीकरण)

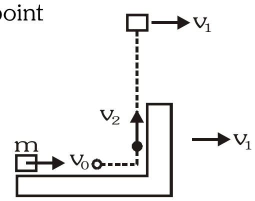
$$T\Delta t = 2m\left(v - \sqrt{\frac{2gx}{3}}\right)$$

$$T\Delta t - T'\Delta t = m\left(v - \sqrt{\frac{2gx}{3}}\right)$$

$$T'\Delta t = m(v-0),\quad v = \sqrt{\frac{3gx}{8}}$$

11. COLM $\Rightarrow mv_0 + 0 = (m + 2m)v_1 \Rightarrow v_1 = \dfrac{v_0}{3}$

After collision at highest point (उच्चतम बिन्दु पर टक्कर के बाद)



$$v_x = 1m/s\left(= \frac{mv_1}{2m}\right)$$

$$v_y = 1m/s\left(= \frac{m\times 2}{2m}\right)$$

COME $\Rightarrow \dfrac{1}{2}mv_0^2 = \dfrac{1}{2}m(v_1^2 + v_2^2) + \dfrac{1}{2}(2m)v_1^2$

$\Rightarrow v_2 = \sqrt{24}$ m/s

Max height attained $= \dfrac{v_2^2}{2g} = 1.2m$

(प्राप्त अधिकतम ऊंचाई)

For the block (ब्लॉक के लिए) $v_x = 1m/s$

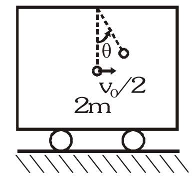
while for the wedge it has (जबकि वेज का वेग)

$v_x = 2m/s$

$$(v_{x_{wedge}} - v_{x_{block}})t = \ell \;\&\; \left(ut + \frac{1}{2}at^2\right)\text{block} = 1.2$$

$\Rightarrow t = 0.4$ sec and $\ell = (2-1)t = 0.4$ m $= 40$ cm

12. COLM : $mv_0 = (M + 2m)\,v_1$



$v_1 = \dfrac{mv_0}{6m} = \dfrac{v_0}{6} = 1m/s$

COME :

$$\Rightarrow \frac{1}{2}(2m)\left(\frac{v_0}{2}\right)^2 = \frac{1}{2}Mv_1^2 + 2m\quad g\quad h + \frac{1}{2}(2m)v_1^2$$

$\Rightarrow h = 0.3\ m = L\,(1 - \cos\theta)$

$\cos\theta = 1 - \dfrac{3}{15} = \dfrac{4}{5} \Rightarrow \theta = 37$

13. For first collision with plate A, final velocity of ball (प्लेट A के साथ प्रथम टक्कर के लिए गेंद का अन्तिम वेग)

$$v_1 = ev_0 = e\sqrt{2gh_0} \qquad \ldots(i)$$

For second collision (द्वितीय टक्कर के लिए)

$$mv = 4mv' \Rightarrow v' = \frac{v}{4}$$

$$\Rightarrow e\sqrt{\frac{2gh_0}{4}} = \sqrt{2gh_2} \Rightarrow e = \frac{2}{3}$$

Height attained after first collision (पहली टक्कर के बाद प्राप्त ऊंचाई)

$$\therefore\ h_1 = e^2h_0 = \frac{2}{3}\times\frac{2}{3}\times 9 = 4m$$

14. Let v = velocity of the ball after collision along the normal (माना v= अभिलम्ब के अनुदिश टक्कर के बाद गेंद का वेग)

J = impulse on ball (गेंद पर आवेग)

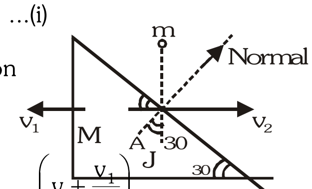
$= v - (-2\cos 30) = v + \sqrt{3}$

Impulse on wedge (वेज पर आवेग)

$J\sin 30 = mv_1 = 2v_1$

$\Rightarrow v = 4v_1 - \sqrt{3}\quad \ldots(i)$

Coefficient of restitution (प्रत्यावस्थान गुणांक)



$$e = -\left(\frac{v_2 - v_1}{u_2 - u_1}\right) \Rightarrow \frac{1}{2} = \frac{\left(v + \frac{v_1}{2}\right)}{2\cos 30°}$$

$$\Rightarrow v = \frac{\sqrt{3}}{2} - \frac{v_1}{2} \qquad \ldots(ii)$$

Solving we get $v_1 = \dfrac{1}{\sqrt{3}}$ m/s

For the ball velocity along incline remains constant. (तल के अनुदिश गेंद का वेग नियत बना रहता है)

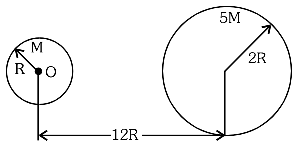
$\therefore\ v' = 2\sin 30 = 1$ m/s

$$\therefore\ \text{Final velocity of ball} = \sqrt{1^2 + \left(\frac{1}{\sqrt{3}}\right)^2} = \frac{2}{\sqrt{3}}\text{ m/s}$$

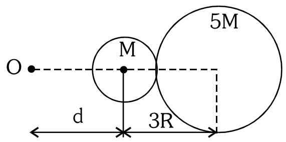
EXERCISE –V-A

1. $v_{cm} = \dfrac{m(2v)+m(-v)}{m+m} = \dfrac{v}{2}$

2. Linear momentum is a vector quantity whereas kinetic energy is a scalar quantity.
(रेखीय संवेग एक सदिश राशि है जबकि गतिज ऊर्जा एक अदिश राशि है)

3. 

Initial position (प्रारम्भिक स्थिति)



Just before collision (टक्कर के ठीक पहले)

For this system, position of centre of mass remains same (इस निकाय के द्रव्यमान केन्द्र की स्थिति वही रहेगी)

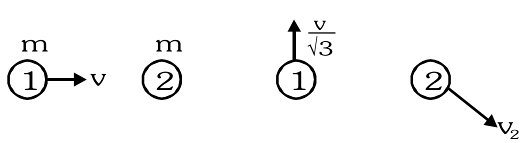
$$\left[\because \vec{F}_{system} = 0\right]$$

$$\frac{M(0)+5M(12R)}{M+5M} = \frac{M(d)+5M(d+3R)}{M+5M} \Rightarrow d = 7.5R$$

4. In order to shift centre of mass, the system must experience an external force, as there is no external force responsible for explosion, hence centre of mass does not shift.
(द्रव्यमान केन्द्र को विस्थापित करने के लिए निकाय पर बाह्य होना चाहिए। यहां विस्फोट के लिए बाह्य बल जिम्मेदार नहीं है, अतः द्रव्यमान विस्थापित नहीं होता है।)

5. Let maximum momentum be p then
(माना अधिकतम संवेग p है तो)

$$\frac{p^2}{2M} = \frac{1}{2}kL^2 \Rightarrow p = L\sqrt{Mk}$$

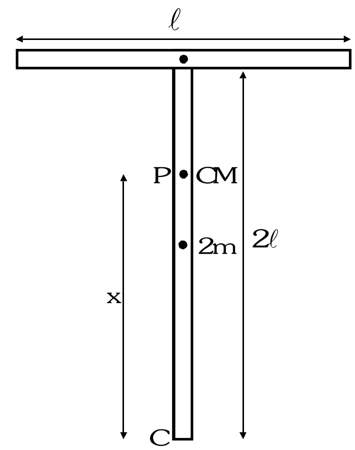
6. 

before collision after collision

From COLM (संवेग संरक्षण से)

$$mv_2 = \sqrt{(mv)^2 + \left(m\frac{v}{\sqrt{3}}\right)^2} \Rightarrow v_2 = \frac{2v}{\sqrt{3}}$$

7. The object will have translation motion without rotation, when $\vec{F}$ is applied at CM of the system.
(वस्तु की बिना घूर्णन के स्थानान्तरणीय गति होगी यदि $\vec{F}$ द्रव्यमान केन्द्र पर लगाया जाये।)



If P is the CM then (यदि P पर द्रव्यमान केन्द्र है तो)

$$m(2\ell - x) = 2m(x-\ell) \Rightarrow x = \frac{4\ell}{3}$$

8. On applying law of conservation of linear momentum (रेखीय संरक्षण नियम द्वारा)

$$\vec{P}_i = \vec{P}_f \Rightarrow 16 \times \vec{0} = 4\vec{v}_4 + 12 \times 4\hat{i} \Rightarrow \vec{v}_4 = 12\left(-\hat{i}\right)$$

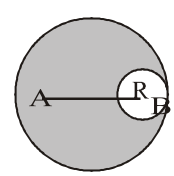
The 4 kg block will move in a direction opposite to 12 kg block with a speed of 12 m/s.The corresponding kinetic energy of 4 kg block
(4 kg का ब्लॉक, 12 kg के ब्लॉक के विपरीत दिशा में 12 m/s के वेग से गति करता है। 4 kg ब्लॉक की गतिज ऊर्जा)

$$= \frac{1}{2} \times 4 \times (12)^2 = 288 \text{ J}$$

9. 

Here $m_1d = m_2x \Rightarrow x = \dfrac{m_1}{m_2}d$

10. Since mass $\propto$ area (चूंकि द्रव्यमान $\propto$ क्षेत्रफल)



Let mass of the bigger disc = 4M

(माना बड़ी चकती का द्रव्यमान)

$\therefore$ mass of the smaller disc = M

(छोटी चकती का द्रव्यमान)

$\therefore$mass of the remaining portion (शेष भाग का द्रव्यमान)

$= 4M - M = 3M$

Now put the cut disc at its place again, centre of mass of the whole disc will be at centre O.

(कटी हुई चकती को पुन: सम्पूर्ण चकती के द्रव्यमान केन्द्र के केन्द्र O पर रखते हैं)

Centre of mass of the smaller disc is at its centre that is at B.

(छोटी चकती का द्रव्यमान केन्द्र, उसके केन्द्र B पर स्थित है)

Suppose CM of the remaining portion is at A and AO is X. Let O as origin

(माना शेष भाग का द्रव्यमान केन्द्र A पर है तथा OA = X है, माना O मूल बिन्दु है)

$$\therefore\ 3M(x)=RM \Rightarrow x = \frac{R}{3}$$

This suggests that centre of mass of remaining disc will shift from the centre of original disc by a distance of (1/3)R towards left.

शेष चकती का द्रव्यमान केन्द्र बांयी ओर (1/3)R दूरी द्वारा मूल चकती के द्रव्यमान केन्द्र से विस्थापित होगा। अत: $\alpha = \frac{1}{3}$

11. Energy loss (ऊर्जा हानि) = $\frac{1}{2}\frac{m_1m_2}{(m_1+m_2)}(u_1-u_2)^2$

$$=\frac{(0.5)(1)}{2[0.5+1]}(2-0)^2 = \frac{2}{3}J$$

12. $$X_{cm} = \frac{\int xdm}{\int dm} = \frac{\int_0^L k\left(\frac{x}{L}\right)^n xdx}{\int_0^L k\left(\frac{x}{L}\right)^n dx} = \left(\frac{n+1}{n+2}\right)L$$

For n=0, $x_{cm} = \frac{L}{2}$ and for $n \to \theta$, $x_{cm} = L$

EXERCISE –V(B)

1. By applying impulse-momentum theorem (आवेग–संवेग प्रमेय लगाने पर)

$$= \left|(m_1\vec{v}_1' + m_2\vec{v}_2') - (m_1\vec{v}_2')\right|$$

$$= \left|(m_1+m_2)\vec{g}(2t_0)\right| = 2(m_1+m_2)gt_0$$

2. Just after collision (टक्कर के तुरन्त बाद)

$$v_c = \frac{10\times14+4\times0}{10+4} = 10\ m/s$$

since spring force is internal force, it cannot change the linear momentum of the (two mass + spring) system. Therefore v_c remains the same.

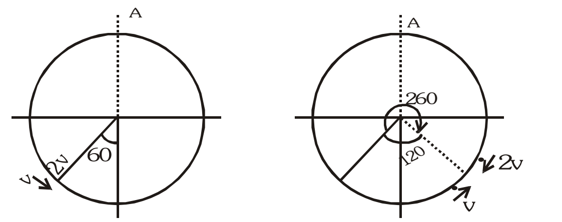
(चूंकि स्प्रिंग बल आन्तरिक बल है। इस निकाय के (दो द्रव्यमान + स्प्रिंग) रेखीय संवेग में कोई परिवर्तन नहीं होता है, अत: v_c समान होगा)

3. $\vec{p}(t) = A[\tilde{i}\cos(kt) - \tilde{j}\sin(kt)]$

$$\vec{F} = \frac{d\vec{p}}{dt} = Ak[-\tilde{i}\sin(kt) - \tilde{j}\cos(kt)]$$

$\vec{F}.\vec{p} = Fp\cos\theta$ But $\vec{F}.\vec{p} = 0 \Rightarrow \cos\theta = 0 \Rightarrow \theta = 90$.

4.



1^{st} collision II^{nd} collision

Particle with velocity 'v' covers and angle of 120 and after collision its velocity become '2v'.

It will cover angle of 240

5. $$Y_{cm} = \frac{m_1y_1+m_2y_2+m_3y_3}{M_1+M_2+M_3}$$

$$Y_{cm} = \frac{6m(0)+m(+a)+m(a)+m(-a)+m(0)}{10m};\ Y_{cm} = \frac{a}{10}$$

Multiple choice questions :

1. As $\vec{p}_1+\vec{p}_2 = \vec{0}$ so $\vec{p}_1'+\vec{p}_2' = \vec{0}$

For (A) $\vec{p}_1'+\vec{p}_2' = (a_1+a_2)\tilde{i} + (b_1+b_2)\tilde{j} + c_1\tilde{k}$

For (B) $\vec{p}_1'+\vec{p}_2' = (a_1+a_2)\tilde{i} + (b_1+b_2)\tilde{j}$

For (C) $\vec{p}_1'+\vec{p}_2' = (c_1+c_2)\tilde{k}$

For (D) $\vec{p}_1'+\vec{p}_2' = (a_1+a_2)\tilde{i} + 2b_1\tilde{j}$

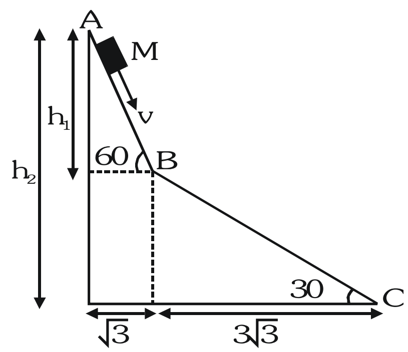
But $a_1, b_1, c_1, a_2, b_2, c_2 \neq 0$

Therefore (A) & (D) is not possible to get

$$\vec{p}_1' + \vec{p}_2' = \vec{0}$$

Comprehension type questions :

1.



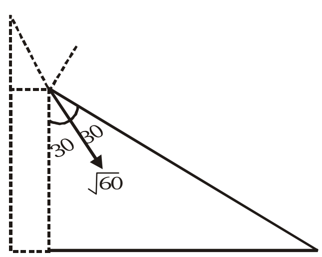
$\dfrac{h_1}{\sqrt{3}}$ = tan60 $\Rightarrow h_1 = 3$m

$\dfrac{h_2 - h_1}{3\sqrt{3}}$ = tan30 $\Rightarrow h_2 - h_1 = 3 \Rightarrow h_2 = 6$m

Velocity of block just before collision at B

(B पर टक्कर के ठीक पहले ब्लॉक का वेग)

$= \sqrt{2gh} = \sqrt{2 \times 10 \times 3} = \sqrt{60}$ ms^{-1}



For totally inelastic collision velocity of block along normal to BC becomes zero and since there is no impulse along BC so momentum (velocity) along BC remains unchanged (पूर्णतया अप्रत्यास्थ टक्कर के लिए BC अभिलम्ब के अनुदिश ब्लॉक का वेग शून्य होगा तथा चूंकि यहां आवेग BC के अनुदिश है अत: BC के अनुदिश संवेग(वेग) अपरिवर्तित रहेगी।)

Speed of block just after collision

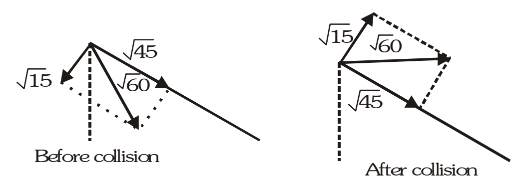
(टक्कर के ठीक बाद ब्लॉक की चाल)

$$v_B = \sqrt{60}\cos 30^\circ = \sqrt{60} \times \frac{\sqrt{3}}{2} = \sqrt{45}\text{ms}^{-1}$$

2. $v_c^2 = v_B^2 + 2g(h_2 - h_1)$

$\Rightarrow v_c = \sqrt{45^2 + 2 \times 10 \times 3} = \sqrt{105}$ ms^{-1}

3.



$\Rightarrow$ vertical component of velocity is zero

(वेग का ऊर्ध्वाधर घटक शून्य है)

Subjective Questions :

1. For body of mass m from A to B

(A से B तक द्रव्यमान m की वस्तु के लिए)

u = 10 m/s (given)

$$a = -\left[\frac{mg\sin\theta + f}{m}\right] = -\left[\frac{mg\sin\theta + \mu mg\cos\theta}{m}\right]$$

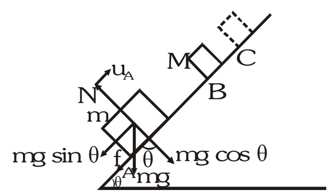
$= -[g \sin\theta + \mu g \cos\theta] = -g [\sin\theta + \mu \cos\theta]$

$= -10\,[0.05 + 0.25 \quad 0.99]$

$= -2.99$ m/s^2

$v^2 - u^2 = 2as$

$\Rightarrow v = \sqrt{100 + 2 - (-2.99) \times 6} = 8$ m/s



After collision (टक्कर के बाद):

Let v_1 be the velocity of mass m after collision and v_2 be the velocity of mass M after collision.Body of mass M moving from B to C and coming to rest.

(माना टक्कर के बाद m द्रव्यमान का वेग v_1 तथा टक्कर के बाद M द्रव्यमान का वेग v_2 है। M द्रव्यमान की वस्तु B से C तक गति करती है और विरामावस्था में आ जाती है)

$u = v_2$; $v = 0$, $a = -2.99$ m/s^2

and s = 0.5 $v^2 - u^2 = 2as$

$\Rightarrow (0)^2 - v_2^2 = 2(-2.99) \quad 0.5$

$\Rightarrow v_2^2 = 1.73$ m/s

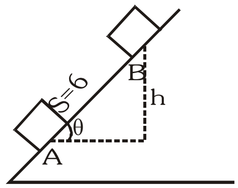
Body of mass m moving from B to A after collision (टक्कर के ठीक बाद m द्रव्यमान की वस्तु B से A गति करती है)

$u = v_1$; $v = +1$ m/s

$(K.E. + P.E.)_{initial} = (K.E. + P.E.)_{final} + W_{frication}$

$$\frac{1}{2}mv_1^2 + mgh = \frac{1}{2}mv^2 + 0 + \mu mgs$$

$\frac{1}{2}v_1^2 + 10 \quad (6 \quad 0.05) = \frac{1}{2}(1)^2 + 0.25 \quad 10 \quad 6$



$v_1 = -5$ m/s

$\sin\theta = \frac{h}{6}$

$h = 6 \sin\theta = 6 \quad 0.05$

$\therefore$ **Coefficient of restitution(प्रत्यावस्थान गुणांक)**

$$e = \left|\frac{\text{Relative velocity of seperation}}{\text{Relative velocity of approach}}\right|$$

$$= \left|\frac{-5-1.73}{8-0}\right| = 0.84$$

2. Consider the vertical motion of the cannon ball (तोप के गोले की ऊर्ध्वाधर गति के लिए)

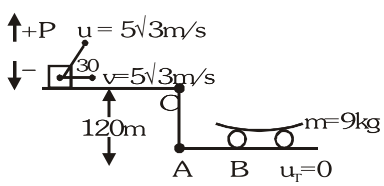
$\because S = ut + \frac{1}{2}at^2 \therefore -120 = 50t_0 - 5t_0^2$

$\Rightarrow 5t_0^2 - 50t_0 - 120 = 0 \Rightarrow t_0^2 - 10\,t_0 - 24 = 0$

$\therefore t_0 = -\frac{(-10) \pm \left[\sqrt{100} - 4(1)(-24)\right]}{2} = 12$ or -2

The horizontal velocity of the cannon ball remains the same (तोप के गोले का क्षैतिज वेग समान होता है)

$\therefore v_x = 100 \cos 30 + 5\sqrt{3} = 55\sqrt{3}$ m/s



$\therefore$ Apply conservation of linear momentum to the cannon ball-trolley system in horizontal direction. If m is the mass of cannon ball and M is the mass of the trolley then (क्षैतिज दिशा में तोप का गोला ट्रॉली निकाय में रेखीय संवेग लगता है यदि तोप का गोले का द्रव्यमान m तथा ट्रॉली का द्रव्यमान M है तो)

$mv_x + M \quad 0 = (m + M)\,V_x \quad \therefore V_x = \frac{mv_x}{m+M}$

where v_x is the velocity of the (cannon ball- trolley) system (जहां v_x निकाय (तोप का गोला–ट्रॉली) का वेग है)

$V_x = \frac{1 \times 55\sqrt{3}}{1+9} = 5.5\sqrt{3}$ m/s

The second ball was projected after 12 second. (दूसरा गोला 12s के बाद प्रक्षेपित होता है)

Horizontal distance covered by the car (कार द्वारा तय क्षैतिज दूरी)

$P = 12 \quad 5\sqrt{3} = 60\sqrt{3}$m

Since the second ball also struck the trolley (चूंकि दूसरा गोला भी ट्रॉली से टकराता है)

$\Rightarrow$ In time 12 seconds the trolley covers a distance of $60\sqrt{3}$ (12 s में ट्रॉली द्वारा तय की गई दूरी $60\sqrt{3}$ होगी)

For trolley in 12 sec (12s में ट्रॉली के लिए)

From

$$s = \left(\frac{u+v}{2}\right)60\sqrt{3} = \left(\frac{5.5\sqrt{3}+v}{2}\right)(12) \Rightarrow v = 7.8 \text{ m/s}$$

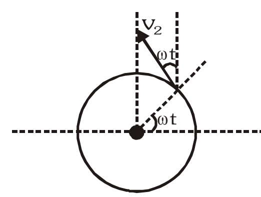
To find the final velocity of the carraige after the second impact we again apply conservation of linear momentum in the horizontal direction (दूसरी टक्कर के बाद गाड़ी का वेग पुनः क्षैतिज दिशा में रेखीय संवेग संरक्षण नियम लगाकर ज्ञात कर सकते हैं)

$m\,v_x + (M + m)7.8 = (M + 2m)\,v_f$

$\therefore 1 \quad 55\sqrt{3} + (9 + 1)7.8 = (9 + 2)v_f$

$\Rightarrow v_f = 15.75$ m/s

3.



$\vec{v}_2 = (-v_2 \sin\omega t\tilde{i} + v_2 \cos\omega t\tilde{j})$ and $\vec{v}_1 = v_1\tilde{j}$

$\vec{v}_{21} = \vec{v}_2 - \vec{v}_1 = -v_2 \sin\omega t\tilde{i} + (v_2 \cos\omega t - v_1)\tilde{j}$

$\vec{p}_{21} = m\vec{v}_{21} = -mv_2 \sin\omega t\tilde{i} + m(v_2 \cos\omega t - v_1)\tilde{j}$

where $\omega = \frac{v_2}{R}$

4. The string snaps and the spring force comes into play. The spring force being an internal force for the two mass-spring system will not be able to change the velocity of centre of mass. This means the location of centre of mass at time t will be v_0 t (जब रस्सी टूटती है तो स्प्रिंग बल ही लगता है। दो द्रव्यमान स्प्रिंग निकाय के लिए स्प्रिंग बल एक आन्तरिक बल होता है जो द्रव्यमान केन्द्र के वेग में कोई परिवर्तन नहीं करता है। समय t पर द्रव्यमान केन्द्र की स्थिति v_0t होगी।)

$x_{cm} = \frac{m_1x_1 + m_2x_2}{m_1 + m_2} = v_0 t$

$\Rightarrow$ $m_1[v_0t-A(1-\cos\omega t)] + m_2x_2 = v_0tm_1 + v_0tm_2$

$\Rightarrow$ $m_2x_2 = v_0tm_1 + v_0tm_2 - v_0tm_1 + m_1A(1-\cos\omega t)$

$\Rightarrow$ $m_2x_2 = v_0tm_2 + m_1A(1-\cos\omega t)$

$\Rightarrow$ $x_2 = v_0t + \frac{m_1}{m_2}A(1-\cos\omega t)$

(b) Given that $x_1 = v_0t - A(1-\cos\omega t)$

$$\therefore \frac{dx_1}{dt} = v_0 - A\omega\sin\omega t$$

$$\therefore \frac{d^2x_1}{dt^2} = -A\omega^2\cos\omega t \ ... \text{(i)}$$

This is the acceleration of mass m_1. When the spring comes to its natural length instantaneously then (यह द्रव्यमान m_1 का त्वरण है जब स्प्रिंग इसकी मूल लम्बाई में आती है तो)

$$\frac{d^2x_1}{dt^2} = 0 \text{ and } x_2 - x_1 = \ell_0$$

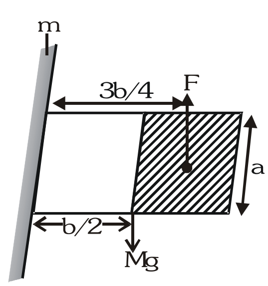
$$\therefore \left[v_0t + \frac{m_1}{m_2}A(1-\cos\omega t)\right] - [v_0t - A(1-\cos\omega t)] = \ell_0$$

$$\left(\frac{m_1}{m_2}+1\right) A(1-\cos\omega t) = \ell_0$$

Also when $\frac{d^2x_1}{dt^2} = 0$; $\cos\omega t = 0$ from (1)

$$\therefore \ell_0 = \left(\frac{m_1}{m_2}+1\right)A$$

5. Since the plate is held horizontal therefore net torque acting on the plate is zero. (चूंकि प्लेट क्षैतिज है अत: प्लेट पर लगने वाला कुल बलाघूर्ण शून्य होगा)



$\Rightarrow$ $Mg\ \frac{b}{2}\ \ F\ \ \frac{3b}{4}$ ... (i)

$F = n\ \frac{dp}{dt}\ \ (\text{Area}) = n\ \ (2mv)\ \ a\ \ \frac{b}{2}$ ... (ii)

From (i) and (ii) $Mg\ \frac{b}{2} = n\ \ (2mv)\ \ a\ \ \frac{b}{2}\ \ \frac{3b}{4}$

$\Rightarrow$ $3\ \ 10 = 100\ \ 2\ \ 0.01\ \ v\ \ 1\ \ \frac{3\times 2}{4}$

$\Rightarrow$ $v = 10$ m/s

6. For collision between A & B

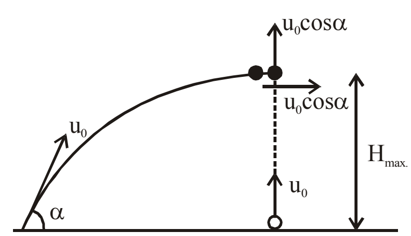
$$v_A = \frac{(m-2m)}{(m+2m)}(9) = -3ms^{-1}$$

$$v_B = \frac{2(m)}{(m+2m)}(9) = +6ms^{-1}$$

For collision between B and C

$$v_C = \left(\frac{2m}{2m+m}\right)(6) = 4ms^{-1}$$

7.



Maximum height of first particle $H_{max} = \frac{u_0^2\sin^2\alpha}{2g}$

Speed of 2^{nd} particle at height H_{max} given as

$v_y^2 = u_0^2 - 2gH_{max} = u_0^2 - u_0^2\sin^2\alpha \Rightarrow v_y = u_0\cos\alpha$

By Momentum Conseravtion

$\vec{p}_f = \vec{p}_i \Rightarrow 2m\vec{v}_f = mv_0\cos\alpha\tilde{i} + mv_0\cos\alpha\tilde{j}$

$\Rightarrow \vec{v}_f = \frac{v_0\cos\alpha}{2}(\tilde{i}+\tilde{j})$

$\Rightarrow$ Angle with horizontal immediately after the collision $= \frac{\pi}{4}$